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Properties of Transpose Conjugate of a Matrix

(i) $(A^*)^* = A$

(ii) $(A + B)^* = A^* + B^*$

(iii) $(kA)^* = kA^*$

(iv) $(AB)^* = B^*A^*$

(V) $(An)^* = (A^*)n$

Some Special Types of Matrices

1. Orthogonal Matrix

A square matrix of order n is said to be orthogonal, if AA’ = I_n = A’A Properties of Orthogonal Matrix

(i) If A is orthogonal matrix, then A’ is also orthogonal matrix.

(ii) For any two orthogonal matrices A and B, AB and BA is also an orthogonal matrix.

(iii) If A is an orthogonal matrix, A^{-1} is also orthogonal matrix.

2. ldempotent Matrix

A square matrix A is said to be idempotent, if $A^2 = A$.

Properties of Idempotent Matrix

(i) If A and B are two idempotent matrices, then

- AB is idempotent, if AB = BA.
- A + B is an idempotent matrix, iff
  AB = BA = 0
- AB = A and BA = B, then $A^2 = A$, $B^2 = B$

(ii)

- If A is an idempotent matrix and A + B = I, then B is an idempotent and AB = BA= 0.
- Diagonal (1, 1, 1, …,1) is an idempotent matrix.
- If I_1, I_2 and I_3 are direction cosines, then

is an idempotent as $|\Delta|^2 = 1$.

A square matrix A is said to be involutory, if $A^2 = I$

4. Nilpotent Matrix

A square matrix A is said to be nilpotent matrix, if there exists a positive integer m such that $A^2 = 0$. If m is the least positive integer such that $A^m = 0$, then m is called the index of the nilpotent matrix A.

5. Unitary Matrix

A square matrix A is said to be unitary, if A’A = I

Hermitian Matrix

A square matrix A is said to be hermitian matrix, if $A = A^*$ or
= a_{ij}, for a_{ji} only.

Properties of Hermitian Matrix

1. If A is hermitian matrix, then kA is also hermitian matrix for any non-zero real number k.

2. If A and B are hermitian matrices of same order, then $\lambda_1 A + \lambda B$, also hermitian for any non-zero real number λ_1, and λ.
3. If A is any square matrix, then AA* and A* A are also hermitian.
4. If A and B are hermitian, then AB is also hermitian, iff AB = BA
5. If A is a hermitian matrix, then A is also hermitian.
6. If A and B are hermitian matrix of same order, then AB + BA is also hermitian.
7. If A is a square matrix, then A + A* is also hermitian,
8. Any square matrix can be uniquely expressed as A + iB, where A and B are hermitian matrices.

Skew-Hermitian Matrix

A square matrix A is said to be skew-hermitian if A* = – A or a_{ji} for every i and j.

Properties of Skew-Hermitian Matrix

1. If A is skew-hermitian matrix, then kA is skew-hermitian matrix, where k is any non-zero real number.
2. If A and B are skew-hermitian matrix of same order, then $\lambda_1 A + \lambda_2 B$ is also skew-hermitian for any real number λ_1 and λ_2.
3. If A and B are hermitian matrices of same order, then AB — BA is skew-hermitian.
4. If A is any square matrix, then A — A^* is a skew-hermitian matrix.
5. Every square matrix can be uniquely expressed as the sum of a hermitian and a skew-hermitian matrices.
6. If A is a skew-hermitian matrix, then A is a hermitian matrix.
7. If A is a skew-hermitian matrix, then A is also skew-hermitian matrix.